Analysis of Dual-Frequency Stacked Patch Antennas Using Subsectional Bases

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Abstract—**The analysis of stacked, microstrip patches for dual-frequency antenna applications is performed by the method of moments applied in the spatial domain and using simple, subsectional, basis functions. The moment matrix is filled, and solved, quickly by exploiting symmetries and performing all computations in four-byte, single precision. The use of subsectional bases enables applications to microstrip structures of arbitrary shapes. Resonant frequencies and reflection coefficients of two stacked patches are well-predicted by linear systems of surprisingly low order.**

I. Introduction

The use of two-layer microstrip structures as dual-frequency antennas or resonators has received much attention recently [1], [2]. Early designs of dual-frequency, stacked patch antennas were based on experiment and sparse empirical data [3]. Other recently reported work includes: a significant improvement in bandwidth from the stacking of narrow microstrip dipoles, with only longitudinal currents considered [4]; extensive results from an experimental study of stacked rectangular patch antennas [5]; and a study of stacked rectangular patches based on the spectral domain method [6].

This letter presents results on analytical and experimental investigations of stacked microstrip patches for dual-frequency applications. Subsectional basis and weighting functions, successfully applied in the case of one-layer microstrip [7], are here applied to two-layer structures [8], in contrast with the work of other authors who have, in general, applied the method of moments to stacked patches by expansion of the unknown surface current on the two conductive patches into a set of entire-domain, or eigen-, functions [1], [2], [6]. Although the orders of the resultant linear systems are generally smaller than those generated using subsectional basis functions, entire-domain modes are limited to regular patch geometries. Subsectional basis functions, on the other hand, are attractive because they can be applied to arbitrary conductor shapes. Furthermore, Toeplitz symmetries can be exploited to reduce dramatically the computation needed to fill the matrix describing the system.

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The work described here is concerned with rectangular patches only as a first step, and in order to facilitate comparison with earlier work. Subsequent work, now in progress, makes use of the full flexibility of this approach in the study of two-layer planar structures of arbitrary shape, including novel microstrip circuit elements, as well as antennas.

II. Approach

The boundary value problem for the unknown surface current is expressed in terms of a mixed potential integral equation [7],

$$\boldsymbol{E}_t^i(\boldsymbol{r}) = Z_s \boldsymbol{J}_s(\boldsymbol{r}) + j\omega \int_s \overleftrightarrow{\boldsymbol{G}}_A(\boldsymbol{r}|\boldsymbol{r}') \cdot \boldsymbol{J}_s(\boldsymbol{r}')ds' - \frac{1}{j\omega}\nabla_t \int_s G_V(\boldsymbol{r}|\boldsymbol{r}') \cdot \nabla_t \boldsymbol{J}_s(\boldsymbol{r}')ds', \quad (1)$$

where $\boldsymbol{E}_t^i$ is the incident electric field tangent to the conductive surfaces and $\boldsymbol{J}_s$ is the unknown surface current.

An approximate solution to (1) is found by the method of moments. As in the one-layer microstrip problem [7], the conductive patches (and feedline) are subdivided into charge cells, over which the unknown current $\boldsymbol{J}_s$ is expanded in a series of subsectional rooftop functions [9]. The weighting functions are one-dimensional pulses, or razor-edges. Proceeding along well-known steps enables (1) to be approximated by a system of linear equations

$$[V] = [Z] \cdot [I], \quad (2)$$

where $[V]$ is the excitation, $[Z]$ is the moment matrix and $[I]$ is the vector of unknown expansion coefficients for the surface current. Evaluation of $[Z]$ involves the convolution of source terms (basis functions) with the observer positions (weighting functions). These operations are equivalent to surface integrals under a coordinate transformation and account for the vast majority of the computational effort.

III. Numerical Implementation

The Green's functions, $\boldsymbol{G}_A$ and G_V in (1), define the spatial impulse response of the two-layer substrate [1]. These are precomputed and stored (i.e., on hard disk) as look-up tables.

Toeplitz symmetries are exploited and, hence, only a subset of the surface integrals previously mentioned need be evaluated. For example, with each rectangular patch divided into $7 \times 3 = 21$ cells, it is easy to show that only 770 of the required 3848 surface integrals are unique (The 3848 figure

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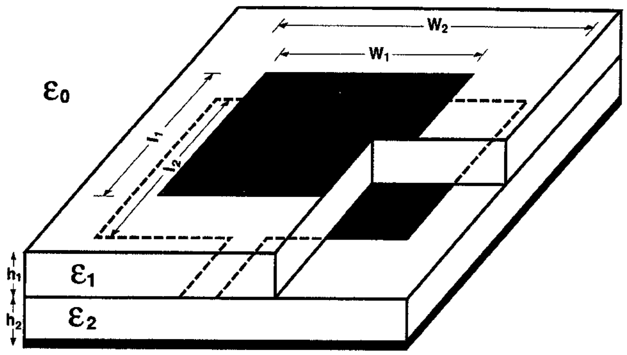

Fig. 1. Two symmetrically situated patches excited by a feedline connected to the lower patch.

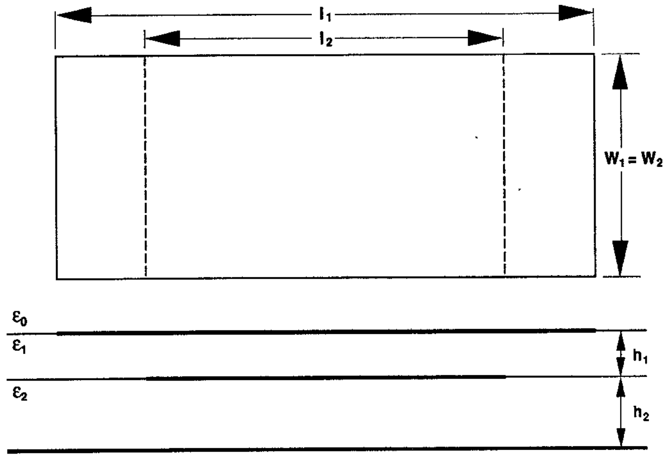

Fig. 2. Dual-frequency structure composed of symmetrically situated patches of equal width.

arises from the 32 rooftop functions for the current on each patch and from the associated 21 pulse functions for the charge density on each patch). Self-terms are evaluated efficiently by subtraction and analytical integration of singularities [8], [10]. The moment matrix is then filled completely with fast integer and logical operations [8].

The resonances of the system are detected by monitoring the determinant of the moment matrix, $[Z]$ in (2), after a transformation to diagonal form using standard Gaussian methods. Input impedance is evaluated by the interpolation of the standing-wave, when a feedline is present [8], [11]. It is worth noting that all computations are performed in single (four byte) precision on a personal computer (286/287 platform @12 MHz), with a typical run for two patches taking about 12 minutes per frequency point.

The software implementation of the analysis of the two-layer structure, Fig. 1, was easily extended to include other structures such as a single patch with a dielectric cover or an electromagnetically-coupled patch.

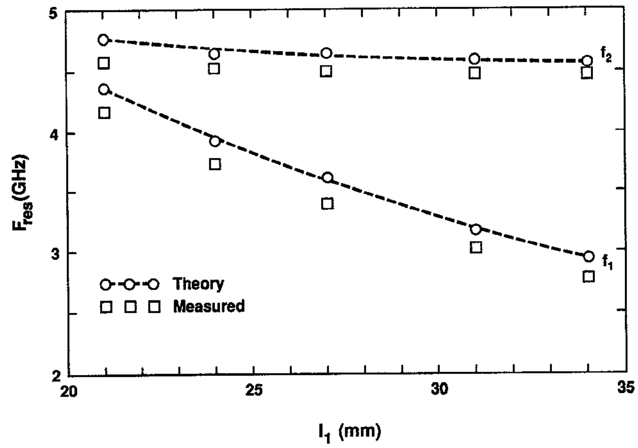

Fig. 3. Dual resonances of symmetrically situated patches, as shown in Fig. 2, as a function of the length (l_1) of the top patch. Dimensions are $w_1 = w_2 = 15$ mm, $l_2 = 21$ mm, $h_1 = 1.6$ mm, $h_2 = 1.8$ mm, $\epsilon_{r1} = \epsilon_{r2} = 2.33$.

IV. Results

The analytical approach previously described is flexible and has been applied to various configurations. Its accuracy has been checked both analytically and by comparison with experiment [8]. Here, we show results for an antenna made of two symmetrically situated patches (Fig. 2) where the resonant frequencies are varied by changing the length of the top patch. Numerically, resonances are detected by locating minima in the determinant of a moment matrix of order 64. This eigenvalue problem was physically implemented by embedding a thin (100 μm) probe in the substrate and below the patches. The result is a proximity feed which excites the structure without undue perturbation of the resonances. Agreement between calculated and measured resonances (Fig. 3) is comparable to that reported elsewhere with multiple entire-domain bases [1], [2]. This similar, recurring offset between calculation and measurement is worthy of further investigation.

The effect of placing two patches in close proximity is exemplified by the results in Table I. Here, the resonant frequencies of two stacked patches are compared to the resonant frequencies of the respective patches with the other patch absent. The resonant frequency of the smaller patch is seen to be much more sensitive to the presence of the larger patch. This reactive loading due to the presence of another patch is well-predicted by the behavior of the moment matrix.

TABLE I
Resonant Frequencies Of Two Patches (in GHz) $w_1 = w_2 = 15$mm

$h_1 = 0.9$ mm, $h_2 = 1.8$ mm, $\varepsilon_{r1} = \varepsilon_{r2} = 2.33$	One Patch Alone		Two Patches Stacked	
	Theory	Measured	Theory	Measured
Upper Patch 31 mm ×15 mm	3.22	3.08	3.22	3.09
Lower Patch 21 mm ×15 mm	4.33	4.33	4.53	4.55

The magnitude of the reflection coefficient, $|S_{11}|$, of dual-frequency, stacked patches with a frequency ratio $f_2/f_1 \approx 1.5$ is plotted in Fig. 4. A feedline, connected to the lower patch as in Fig. 1, is included in the structure. $|S_{11}|$ calculations are based on an efficient interpolation of the standing-wave pattern along the feedline [8]. The same basis functions are used over

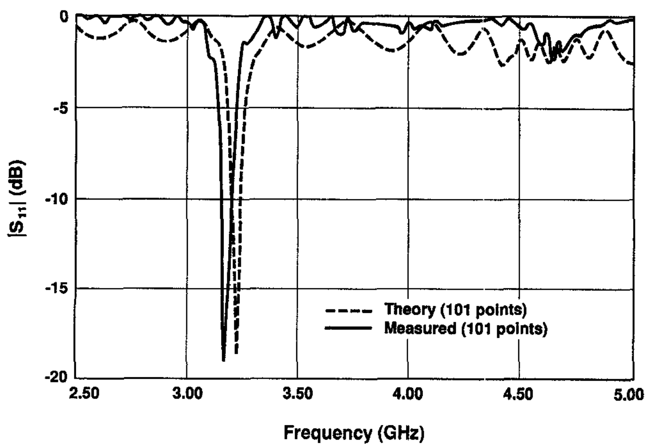

Fig. 4. Magnitude of the return loss as a function of frequency for symmetrically situated patches excited by a feedline connected to the lower patch as shown in Fig. 1. Dimensions are given in Table I. The feedline is 1 mm wide.

the entire frequency range and the resultant moment matrix is of order 140. From the initial results shown in Fig. 4, it is clear that the structure is well-matched at the lower resonant frequency only.

V. Conclusion

The analysis of dual-frequency, stacked, microstrip patch antennas has been shown to be feasible employing simple subsectional bases in the spatial domain implementation of the method of moments. Good results are possible with relatively few basis functions and single precision computation on a personal computer. The analysis predicts the shift in the fundamental resonances of stacked patches in close proximity as well as yielding good results for the return loss of stacked patches fed by a microstrip line. The approach is flexible and can be applied to stacked microstrip structures of arbitrary conductor shapes. The study of novel stacked structures is ongoing in concert with further improvements in the computational scheme.

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